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AN INVENTORY MODEL FOR NON-INSTANTANEOUS DETERIORATING ITEMS WITH PARTIAL BACKLOGGING AND STOCK-DEPENDENT DEMAND

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ABSTRACT

In this paper, after study other similar research we will discuss a stock dependent deteriorating inventory model with partial backlogging and optimal replenishment policy. The necessary and sufficient conditions of the existence and uniqueness of the optimal solution are shown. The time at which demand rate changes, may be deterministic or uncertain. Finally, numerical example is presented to demonstrate the developed model and the solution procedure.

KEYWORDS: Inventory, Non-Instantaneous Deterioration, Stock-Dependent Demand, Partial Backlogging

1. INTRODUCTION

The purpose of this paper is to analyse an inventory model for a deteriorating item with stock dependent demand under permissible delay in payment. This model is based on electronics and manufacturing items. In this case results occurs in asymphtotic form. This model based on stock-dependent, holding and shortage cost.

Wee(24) considered stock-dependent demand rate in a periodic reviewinventory system. Subbaiah et al.(21) and Rao et al.(16) developed an inventory model with stock-dependent demand.

The rest of our work is organized as follows. In section 2, the notation and assumptions are given. In section 3, we present the mathematical model. In section 4, numerical examples are presented to demonstrate the model. At last, we conclude in section 5.

2. MODEL FORMULATION

The proposed model is explored under the same assumptions as adopted by S. R. Singh & A. K. Malik (14), except the ones related to the stock dependent demand, the inflation and time discounting.

The inventory system is governed by the following differential equations in the interval (0, T) are

$$\frac{dI_{R1}(t)}{dt} = -[a + bI_{R1}(t)] \quad 0 \le t \le t_1 \tag{1}$$

$$\frac{dI_{R2}(t)}{dt} + \alpha I_{R2}(t) = -[a + bI_{R2}(t)] \quad t_1 \le t \le t_2 \tag{2}$$

$$\frac{dI_{01}(t)}{dt} = 0 \quad 0 \le t \le t_1 \tag{3}$$

$$\frac{dI_{02}(t)}{dt} + \beta I_{02}(t) = 0 \quad t_1 \le t \le t_2 \tag{4}$$

$$\frac{dI_{03}(t)}{dt} + \beta I_{03}(t) = -[a + bI_{03}(t)] \quad t_2 \le t \le t_3 \tag{5}$$

$\frac{dI_N(t)}{dt}$ = - $(a+bt)e^{-\delta t}$ t_3≤t≤T (6)

With the boundary conditions $I_{R1}(0)$=R, $I_{R2}(t_2)$ =0, $I_{01}(t_1)$ =W, $I_{03}(t_3)$ =0

$I_N(t_3)$ = 0 respectively, solving these differential equations, we get the inventory level as follows:

$I_{R1}(t) = Re^{-bt} - \frac{a}{b}\{1 - e^{-bt}\}$, $0 \le t \le t_1$ (7)

$I_{R2}(t) = \{e^{(b+\alpha)(t_2-t)} - 1\}$, $t_1 \le t \le t_2$ (8)

$I_{01}(t)$ =W, $0 \le t \le t_1$ (9)

$I_{02}(t)$= W$e^{\beta(t_1-t)}$ $t_1 \le t \le t_2$ (10)

$I_{03}(t) = \frac{a}{b+\beta}\{e^{(b+\beta)(t_3-t)} - 1\}$, $t_2 \le t \le t_3$ (11)

$I_N(t) = \frac{a}{\delta}\left(e^{-\delta t} - e^{-\delta t_3}\right) + \frac{b}{\delta}(te^{-\delta t} - t_3 e^{-\delta t_3}) + \frac{b}{\delta^2}(e^{-\delta t} - e^{-\delta t_3})$, $t_3 \le t \le T$ (12)

Putting t=T in equation (12), we obtain the maximum amount of demand backlogged per cycle as follows:

S = - $I_N(T)$ = - $\frac{a}{\delta}\left(e^{-\delta T} - e^{-\delta t_3}\right) + \frac{b}{\delta}\left(Te^{-\delta T} - t_3 e^{-\delta t_3}\right) + \frac{b}{\delta^2}(e^{-\delta T} - e^{-\delta t_3})$ (13)

Considering continuity of $I(t)$at t= t_1, it follows from equations (7) and (8) that

$I_{R1}(t_1)$ = $I_{R2}(t_1)$

R=$\frac{a}{b+\alpha}\{e^{(b+\alpha)(t_2-t_1)} - 1\}e^{bt_1} + \frac{a}{b}\{e^{bt_1} - 1\}$ (14)

Substituting equation (14) into (7), we get

$I_{R1}(t)$=$\frac{a}{b+\alpha}\{e^{(b+\alpha)(t_2-t_1)} - 1\}e^{-b(t-t_1)} + \frac{a}{b}\{e^{-b(t-t_1)} - 1\}$, $0 \le t \le t_1$ (15)

Now equation (13) and (14), we can obtain the order quantity, Q; as

Q = R + S

=$\frac{a}{b+\alpha}\{e^{(b+\alpha)(t_2-t_1)} - 1\}e^{bt_1} + \frac{a}{b}\{e^{bt_1} - 1\} + \frac{a}{\delta}\{e^{-\delta T} - e^{-\delta t_3}\} + \frac{b}{\delta}\{Te^{-\delta T} - t_3 e^{-\delta t_3}\} + \frac{b}{\delta^2}\{e^{-\delta T} - e^{-\delta t_3}\}$ (16)

According to given conditions at

t=t_2, $I_{02}(t_2) = I_{03}(t_2)$

$t_3 = t_2 + \frac{1}{b+\beta} log\left\{1 + \frac{w(b+\beta)}{a} e^{\beta(t_1-t_2)}\right\}$ (17)

Next, the total relevant cost per cycle consists of the following elements:

Ordering cost percycle is A. (18)

Inventory holding cost per cycle in **RW** (which is denoted by IHC_{RW} is given by

IHC_{RW}= $C_{HR}\left\{\int_0^{t_1} I_{R1}(t)dt + \int_{t_1}^{t_2} I_{R2}(t)dt\right\}$

$$=C_{HR}\left[\frac{a}{b+\alpha}\{e^{(b+\alpha)(t_2-t_1)}-1\}\{e^{bt_1}-1\}+\frac{a}{b^2}\{e^{bt_1}-bt_1-1\}+\frac{a}{(b+\alpha)^2}\{e^{(b+\alpha)(t_2-t_1)}-(t_2-t_1)(b+\alpha)-1\}\right] \tag{19}$$

Inventory holding cost per cycle in **OW**(which is denoted by IHC_{OW})is given by

$$IHC_{OW}=\left\{\int_0^{t_1}I_{01}(t)dt+\int_{t_1}^{t_2}I_{02}(t)dt+\int_{t_2}^{t_3}I_{03}(t)\,dt\right\}$$

$$=C_{HO}\left[Wt_1+\frac{W}{\beta}\{1-e^{-\beta(t_2-t_1)}\}+\frac{a}{(b+\beta)^2}\{e^{(b+\beta)(t_3-t_2)}-(t_3-t_2)(b+\beta)-1\}\right] \tag{20}$$

Deterioration cost per cycle in **RW** (which is denoted byDC_{RW})is given by

$$DC_{RW}=C_{DR}\left\{I_{R2}(t_1)-\int_{t_1}^{t_2}D(t)dt\right\}$$

$$=C_{DR}\left[\frac{a\alpha}{b+\alpha}\{e^{(b+\alpha)(t_2-t_1)}-(t_2-t_1)(b+\alpha)-1\}\right] \tag{21}$$

Deterioration cost per cycle in **OW**(which is denoted by DC_{OW}) is given by

$$DC_{OW}=C_{DO}\left\{I_{02}(t_1)-\int_{t_1}^{t_2}I_{02}(t)dt-\int_{t_2}^{t_3}D(t)dt\right\}$$

$$=C_{DO}\left[W+\frac{W}{\beta}\left[1-e^{-\beta(t_2-t_1)}\right]-a(t_3-t_2)+\frac{ab}{(b+\beta)^2}\{1-e^{(b+\beta)(t_3-t_2)}+(t_3-t_2)(b+\beta)\}\right] \tag{22}$$

Shorting cost per cycle due to **backlog** (which is denoted by SC) is given by

SC = $C_S\int_{t_3}^{T}[-I_N(T)e^{-RT}]\,dT$

$$=\text{-}C_S\left[\frac{a}{\delta}\left(-\frac{e^{-\delta t_3}(e^{-RT}-e^{-Rt_3})}{R}+\frac{(e^{-(\delta+R)T}-e^{-(\delta+R)t_3})}{(\delta+R)}\right)+\frac{b}{\delta}\left(-\frac{t_3e^{-\delta t_3}(e^{-RT}-e^{-Rt_3})}{R}+\frac{Te^{-(\delta+R)T}-t_3e^{-(\delta+R)t_3}}{\delta+R}+\right.\right.$$

e–δ+RT–e–δ+Rt3δ+R2+bδ2–e–δt3e–RT–e–Rt3R+ *e–(δ+R)T–e–δ+Rt3(δ+R)*

(23)

Opportunity cost per cycle due to **lost sales**(which is denoted by OC)is given by

OC = $C_O\int_{t_3}^{T}(a+bT)\,(1-e^{-\delta T})e^{-RT}dT$

$$=C_0\left[\frac{ae^{-Rt_3}}{R}-\frac{ae^{-RT}}{R}+\frac{bt_3e^{-Rt_3}}{R}-\frac{bTe^{-RT}}{R}+\frac{be^{-Rt_3}}{R^2}-\frac{be^{-RT}}{R^2}-\frac{ae^{-(\delta+R)t_3}}{\delta+R}+\frac{ae^{-(\delta+R)T}}{(\delta+R)}-\frac{bt_3e^{-(\delta+R)t_3}}{(\delta+R)}+\frac{bTe^{-(\delta+R)T}}{\delta+R}-\right.$$

be–(δ+R)t3δ+R2+be–(δ+R)Tδ+R2

(24)

Therefore, the total relevant inventory cost per unit time is given by

$$\text{TC}(t_1,t_2,T)\text{=(1/T)[A+}IHC_{RW}+IHC_{OW}+DC_{RW}+DC_{OW}+SC+OC] \tag{25}$$

Substituting Equation (18)-(24) in the above equation (25), we get

$$\text{TC}(t_1,t_2,T)=\frac{a}{T}\left\{\frac{A}{a}+\left[C_{HR}\left(e^{bt_1}-1+\frac{1}{b+\alpha}\right)+\alpha C_{DR}\right]\left[\frac{1}{b+\alpha}\{e^{(b+\alpha)(t_2-t_1)}-1\}\right]-(t_2-t_1)\left(\alpha C_{DR}+\frac{C_{HR}}{b+\alpha}\right)+\right.$$
$$\left.\frac{C_{HR}}{b^2}\left[e^{bt_1}-bt_1-1\right]\right\}+\left[\frac{(C_{HO}-bC_{DO})}{(b+\beta)^2}\{e^{(b+\beta)(t_3-t_2)}-(t_3-t_2)(b+\beta)-1\}\right]+\frac{W}{a}(t_1C_{HO}+C_{DO})-C_{DO}(t_3-$$
$$t_2)-C_S\left[\frac{a}{\delta}\left\{\frac{-e^{-\delta t_3}(e^{-RT}-e^{-Rt_3})}{R}+\frac{e^{-(\delta+R)T}-e^{-(\delta+R)t_3}}{(\delta+R)}\right\}+\right.$$

$$\frac{b}{\delta}\left\{\frac{-t_3e^{-\delta t_3}(e^{-RT}-e^{-Rt_3})}{R}+\frac{(Te^{-(\delta+R)T}-t_3e^{-(\delta+R)t_3})}{(\delta+R)}+\frac{e^{-(\delta+R)T}-e^{-(\delta+R)t_3}}{(\delta+R)^2}+\frac{b}{\delta^2}\left(-\frac{e^{-\delta t_3}(e^{-RT}-e^{-Rt_3})}{R}+\frac{(e^{-(\delta+R)T}-e^{-(\delta+R)t_3})}{(\delta+R)}\right\}\right]+$$
$$C_O\left[\frac{a}{R}(e^{-Rt_3}-e^{-RT})+\frac{b}{R}(t_3e^{-Rt_3}-Te^{-RT})+\frac{b}{R^2}(e^{-Rt_3}-e^{-RT})+\frac{a}{(\delta+R)}\left(e^{-(\delta+R)T}-e^{-(\delta+R)t_3}\right)+\frac{b}{(\delta+R)}\left(Te^{-(\delta+R)T}-\right.\right.$$
$$\left.\left.t_3e^{-(\delta+R)t_3}\right)+\frac{b}{(\delta+R)^2}\left(e^{-(\delta+R)T}-e^{-(\delta+R)t_3}\right)\right]$$

The total relevant inventory cost per unit time is minimum if

$$\frac{\partial TC}{\partial t_1}=0,\frac{\partial TC}{\partial t_2}=0,\frac{\partial TC}{\partial T}=0,$$

$$\frac{\partial^2 TC}{\partial t_1^2},\frac{\partial^2 TC}{\partial t_2^2},and\,\frac{\partial^2 TC}{\partial T^2}\,allarepositive.$$

NUMERICAL EXAMPLE

In order to illustrate the above solution procedure, consider an inventory system with the following data:

A=250;$C_{HR}=.5, C_{HO}=.6, C_{DR}=1.5, C_{DO}=1.6, C_S=2.5, C_O=2, \delta=2, a=600, b=0.1; \alpha=0.08, \beta=$ .09 $and t_1=1/12=0.0833, t_2=1/8=0.125.$

CONCLUSIONS

In this paper a stock dependent deteriorating inventory model with partial backlogging and replenishment number, the minimum present value of total relevant cost. The nature of demand electronics and manufacturing items is increasing-steady-decreasing. The demand pattern assumed here is found to occur not only for all types of seasonal products but also for fashion apparel, computer chips of advanced computers, spare parts, mobiles, circuit, toys, food items etc. Shortages are allowed and the backlogging rate is variable and dependent on the waiting time for the next replenishment. Finally, numerical examples on this model highlighting the results.

Table 1

M	T_1	T_2	Q	Tc
1	.083333	.125	–3122.97	–5627427
2	.166667	.25	–2878.31	–5174274
3	.25	.375	–2642.04	–5387167
4	.33333	.5	–2411.62	–6369530
5	.416667	.625	–2185.02	–8295696
6	.5	.75	–1960.69	–1.1E+07
7	.583333	.875	–1737.37	–1.6E+07
8	.666667	1	–1514.12	–2.3E+07
9	.75	1.125	–1290.15	–3.2E+07

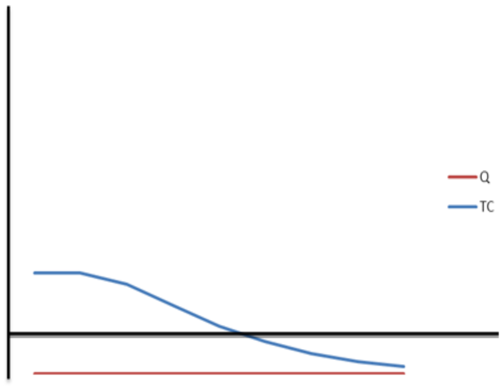

Figure 1: Graphical Representation of Inventory System

REFERENCES

1. Baker, R. C, and Urban, T. L, "A deterministic inventory system with an inventory-level dependent demand rate", J.Opl.Res.Soc, 39 (9) (1988) 823-831.
2. Basu, M, and Sinha, S, "An ordering policy for deteriorating ites with two Component demand and price breaks allowing shortages", Opsearch, 44, 1 (2007).
3. Chakrabarti, T, and Sen, S, "An order inventory model with variable rate of Deteriorating and alternating replenishing rates considering shortage", Journal of Operations Research Society of India, 44, 1 (2007) 17-26.
4. Chakrabarti, T, and Sen, S, "An EOQ model for deteriorating items with quadratic time varying demand and shortages in all cycles", Journal of Mathematics and System Sciences, 44(1) (2008) 141-148.
5. Chung, K. J, and Hwang, Y. F, "The optimal cycle time for EOQ inventory model under permissible delay in payments", International Journal of Production Economics, 84 (2003)307-318.
6. Dye, C. Y, "A deteriorating inventory model with stock dependent demand and partial backlogging under conditions of permissible delay in payments", Opsearch, 39(2002) 3 & 4.
7. Gupta, R, and Vrat, P, "Inventory model for stock-dependent consumption rate", Opsearch, 23 (1) (1986) 19-24.
8. Ghare, P. M, and Scharder, G. F, "A model for exponentially decaying inventory'', J. Indust. Engg, 14 (1963) 238-243.
9. Jamal, A. M. M, Sarkar, B. R, and Wang, S, "An ordering policies for deteriorating items with allowable shortages and permissible delay in payment", J.Oper. Res.Soc, 48 (1997) 826-833.
10. Khanra, S, Sana, S. S, and Chaudhuri, K. S, "An EOQ model for perishable item with stock and price dependent demand rate", International Journal of Mathematics in Operational Research, 2 (3) (2010) 320-335.
11. Levin, R. I, Melaughlin, C. P, Lamone, R. P, and Kottas, J. F, Production/Operations Management: Contemporary Policy for Managing Operating Systems, Mc Graw-Hill, New York, (1972) 373.
12. Mandal, B. N, and Phaugdar, S, "A note on an inventory model with stock-dependent consumption rate", Opsearch, 26 (1) (1989) 43-46.
13. Mandal, B. N, and Phaujdar, S, "An inventory model for deteriorating items and stock dependent consumption rate", J.Opl. Res. Soc, 40 (5) (1989) 483-488.
14. Malik and Malik, "An inventory model with stock –dependent demand with two storages capacity for non-instantaneous deteriorating items, Yugoslav Journal of Operations Research, Vol. 1 (2013) 1255-1260.
15. Pal, A. K, and Mandal, B, "An EOQ model for deteriorating inventory with alternating demand rates", Korean J

Cop & Appl Math, 4 (2) (1997) 397-407.

16. Rao, K. S, and Sarma, K. V. S, “An order-level inventory model under L2 system with power pattern demand”, International Journal of Management and Systems, 20 (2004) 31-40.
17. Rau, H, Wu, M. Y, and Wee, H. M, “Deteriorating item inventory model with shortage due to supplier in an integrated supply chain”, International Journal of System Science, 35 (5) (2004) 293-303.
18. Reddy, G. S. N, and Sarma, K. V. S, “A periodic review inventory problem with variable stock dependent demand”, Opsearch, 38 (3) (2001) 332-341.
19. Silver, E. A, “Operations research in inventory management”, Opns. Res, 29 (1981) 628-645.
20. Silver, E. A, and Peterson, R, Decision Systems for Inventory Management and Production Planning, 2nd edn. Wiley, New York, 1985.
21. Subbaiah, K. V, Rao, K. S, and Satyanarayana, B, “Inventory models for perishable items having demand rate dependent on stock level”, Opsearch, 41 (4) (2004) 223-236.
22. Skouri, K, Papachritos, S, “Optimal stoping and restarting production times for an EOQ model with deteriorating items and time-dependent partial backlogging”, International Journal of Production Economics, 81-82 (2003) 525-531.
23. Teng, J. T, and Chang, C. T, “ Economic production quantity models for deteriorating items with price and stock dependent demand”, Computers and Operational Research, 32 (2005)297-308.
24. Wee, H. M, “A replenishment policy for items with a price dependent demand and varying rate of deterioration”, Productio planning and Control, 8 (5) (1997) 494-499.